Global Journal of Science Frontier Research
Mathematics & Decision Sciences
Volume 12 Issue 1 Version 1.0 January 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Fourth-Order Four Point Sturn-Liouville Boundary Value Problem With Non homogeneous Conditions

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Mathematics Subject Classification: 34B10

FOURTH-ORDER FOUR POINT STURN-LIOUVILLE BOUNDARY VALUE PROBLEM WITH NON HOMOGENEOUS CONDITIONS

Strictly as per the compliance and regulations of :

Fourth-Order Four Point Sturn-Liouville Boundary Value Problem With Non homogeneous Conditions

Djibibe Moussa Zakari[α], Tcharie Kokou[Ω]

Abstract - In this paper, the sucient conditions are given for the existence and uniqueness of solutions of the following nonlinear Sturn-Liouville boundary value problem with non homogeneous four point boundary conditions :

$$\begin{cases} u^{(4)}(t) = f(t, u(t), u''(t)), \quad 0 < t < 1 \\ u'(0) - h_0 u(0) = \alpha_0 \\ u'(1) - h_1 u(1) = \alpha_1 \\ a_1 u^{(3)}(t_1) - b_1 u''(t_1) = \lambda_1 \\ a_2 u^{(3)}(t_2) + b_2 u''(t_2) = \lambda_2 \end{cases}$$

where $0 \leq t_1 \leq t_2 \leq 1$ and λ_1 et λ_2 are nonegative parameters.
The dependence continue of the solution on the parameters λ_1 and λ_2 is also investigated.
Keywords and phrases : Fourth-order, Four points, Sturn-Liouville, Boundary value problem, Nonhomogeneous, Cone, Concave, Fixed point, Green's function, Dontinuous dependence, Positive solution.

I. Introduction

Multi-point boundary value problems for ordinary differential equations arise in variety of areas of applied biologics, chemics, mathematics and physics have been studied. For details, see for exemple, [1], [4] -[11] and referencs therein.

In particular, in a recent article [4], Sun and Wang studied a four-point boundary value problem of the form

$$\begin{aligned} & u^{(4)}(t) = f(t, u(t)), \ 0 < t < 1 \\ & \alpha u(0) - \beta u'(0) = \gamma u(1) + \delta u'(1) = 0 \\ & a u''(\xi_1) - b u'''(\xi_1) = -\lambda, \ c u''(\xi_1) + d u'''(\xi_1) = -\mu \end{aligned}$$

In [2], Kong and Kong, investigated following multi-point boundary value problem :

$$\begin{aligned} & u''(t) + a(t) f(u) = 0 \\ & u(0) = \sum_{i=1}^{m} a_i u(t_i) + \lambda, \ u(1) = \sum_{i=1}^{m} b_i u(t_i) + \mu \end{aligned}$$

where λ and μ are nonegative parameter. They derived some conditions for the above boundary value problems to have a unique solution and then studied the dependence of this solution on the parameters λ and μ.

In another paper [5], Ricardo and Luis :

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$$u^{(4)}(t) = f(t, u(t), u''(t)), \ 0 < t < 2\pi$$
$$u(0) = u(2\pi), \ u'(0) = u'(2\pi)$$
$$u''(0) = u''(2\pi), \ u'''(0) = u'''(2\pi)$$
$$u(0) = u(\pi) = u''(0) = u''(\pi) = 0$$

In the present paper, being inspired by [2] and [5], we investigated the fourth - order differential equation

$$u^{(4)}(t) = f(t, u(t), u''(t)), \ 0 < t < 1 \tag{1.1}$$

and the four-point nonhomogeneous Sturn-Liouville boundary conditions

$$u'(0) - h_0 u(0) = \alpha_0 \tag{1.2}$$
$$u'(1) - h_1 u(1) = \alpha_1 \tag{1.3}$$
$$a_1 u^{(3)}(t_1) - b_1 u''(t_1) = \lambda_1 \tag{1.4}$$
$$a_2 u^{(3)}(t_2) + b_2 u''(t_2) = \lambda_2 \tag{1.5}$$

where $0 \le t_1 \le t_2 \le 1$ and λ_1 and λ_2 are nonnegative parameters.

We investigated the existence, uniqueness and parameter dependence continuous solution of the problem (1.1) -(1.5).

We will suppose the following conditions are satisfied :

Conditions 1.1

$\alpha_0, \ \alpha_1, \ h_0, \ h_1, \ a_1, \ b_1$ are nonegative constants and $a_2, \ b_2$ negative constants such that

$$a = -h_0 + h_1 + h_0 h_1 > 0 \ \mathit{and} \ b = b_1 b_2 (t_1 - t_2) - a_1 b_2 - a_2 b_1 > 0.$$

Conditions 1.2

$$h_1 \alpha_0 - h_0 \alpha_1 > 0, \quad \alpha_0 - \alpha_1 - h_1 > 0, \quad a_1 - b_1 t_1 > 0, \quad -b_2 t_2 - a_2 > 0.$$

Conditions 1.3

- $f \ : \ [0, \ +\infty[\times [0, \ +\infty[\times \mathbb{R} \longrightarrow [0, \ +\infty[$ is continuous and monotone increasing in u and u''.
- There exists $0 \le r < 1$, such that : $k^r f(t, u(t), u'') \le f(t, ku(t), ku''(t))$ for all $t \in (0,1)$, and $k \in (0,1)$.

II. Preliminaries and Some Basic Lemmas

Definition 1

Let $\mathbb{E}$ be a reel Banach space with a norm $||.||_{\mathbb{E}}$ *and* K a nonempty closed convex set of $\mathbb{E}$.

1. K is said to be cone if $\alpha K \subseteq \mathbb{E}$ for all $\alpha \ge 0$ and $K \cap (-K) = \{0_{\mathbb{E}}\}$.
2. Every cone K in $\mathbb{E}$ definies a partial ordering in $\mathbb{E}$ by $x \le y \Longleftrightarrow x - y \in K$.
3. A cone K is said to be normal if there existe $\lambda > 0$ such that

$$0 \le x \le y \Longrightarrow ||x||_{\mathbb{E}} \le \lambda ||y||_{\mathbb{E}}.$$

4. A cone K is said to be solid if the interior $\overset{\circ}{K}$ of K is nonempty.
5. An operator $A \ : \ \overset{\circ}{K} \longrightarrow \overset{\circ}{K}$ is called r-concave if

$$k^r A(u) \le A(ku) \ \text{ for all } \ 0 \le k \le 1, \ u \in \overset{\circ}{K},$$

where K *is a solid cone and* $0 \leq r < 1$.

Lemma 2.1

Let E be u Banach space , K be a normal solid cone in E, $0 \leq r < 1$ and $A : \overset{\circ}{K} \longrightarrow \overset{\circ}{K}$ is a r_ concave increasing operateur. Then A has a unique fixed point in $\overset{\circ}{K}$.

proof

The proof of this lemma 2.1 is the same as [1]

Lemma 2.2

Suppose $a \neq 0$ and $b \neq 0$. If $g(t) \in C([0,\ 1])$ and $g(t) \geq 0$ on $[0,\ 1]$, the nonhomogeneous boundary value problem :

$$\begin{aligned} &u^{(4)}(t) = g(t),\ 0 < t < 1 \\ &u'(0) - h_0 u(0) = \alpha_0, \quad u'(1) - h_1 u(1) = \alpha_1 \\ &a_1 u^{(3)}(t_1) - b_1 u''(t_1) = \lambda_1, \quad a_2 u^{(3)}(t_2) - b_2 u''(t_2) = \lambda_2 \end{aligned}$$

has a unique solution

$$u(t) = \int_0^1 K_1(t,x) \int_{t_1}^{t_2} K_2(x,y) g(y)\, dy\, dx + h(t) + \lambda_1 \varphi_1(t) + \lambda_2 \varphi_2(t), \quad 0 \leq t \leq 1$$

where

$$K_1(t,x) = \begin{cases} \dfrac{(1+h_0 x)(1+h_1(1-t))}{a}, & 0 \leq x \leq t \leq 1 \\[2ex] \dfrac{(1+h_0 t)(1+h_1(1-x))}{a}, & 0 \leq t \leq x \leq 1 \end{cases}$$

$$K_2(x,y) = \begin{cases} \dfrac{(b_1(y-t_1)+a_1)(b_2(x-t_2)-a_2)}{b}, & y \leq x,\ t_1 \leq y \leq t_2 \\[2ex] \dfrac{(b_1(x-t_1)+a_1)(b_2(y-t_2)-a_2)}{b}, & x \leq y,\ t_1 \leq x \leq t_2 \end{cases}$$

$$h(t) = \frac{(h_1\alpha_0 - h_0\alpha_1)t}{a} + \frac{\alpha_0 - \alpha_1 - h_1}{a}, \quad 0 \leq t \leq 1$$

$$\varphi_1(t) = \frac{1}{b}\int_0^1 (b_2(x-t_2) - a_2) K_1(t,x)\, dx, \quad 0 \leq t \leq 1$$

$$\varphi_2(t) = \frac{1}{b}\int_0^1 (b_1(x-t_1) + a_1) K_1(t,x)\, dx, \quad 0 \leq t \leq 1$$

Proof

Putting $u''(t) = w(t),\ 0 \leq t \leq 1$.

By vertue of boundaries conditions (1.2)-(1.5), we obtain two following Sturn-Liouville boundary value problems :

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$$(P_1)\ :\ \begin{cases} u''(t) = w(t),\ 0 < t < 1 \\ u'(0) - h_0 u(0) = \alpha_0 \\ u'(1) - h_1 u(1) = \alpha_1 \end{cases} \quad \text{and} \quad (P_2)\ :\ \begin{cases} w''(t) = g(t),\ 0 < t < 1 \\ a_1 w'(t_1) - b_1 w(t_1) = \lambda_1 \\ a_2 w'(t_2) - b_2 w(t_2) = \lambda_2 \end{cases}$$

The Green's fonctions for Sturn-Liouville problems $(P_1$ and (P_2) are respectively K_1 and K_2.

Then the solutions of the boundary value problems (P_1) and (P_2) are :

$$u(t) = -\int_0^1 K_1(t,x)w(x)\,dx + \frac{(h_1\alpha_0 + h_0\alpha_1)t}{a} + \frac{\alpha_1 - (h_1+1)\alpha_0}{a} \tag{2.1}$$

$$w(t) = -\int_{t_1}^{t_2} K_2(t,y)g(y)\,dy + \frac{(b_2(x-t_2)-a_2)\lambda_1}{b} + \frac{(b_1(x-t_1)+a_1)\lambda_2}{b} \tag{2.2}$$

Substituting (2.2) into (2.1), we get

$$u(t) = \int_0^1 K_1(t,x)\int_{t_1}^{t_2} K_2(x,y)g(y)\,dy dx + h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t), \quad 0 \le t \le 1.$$

This completes the proof of Lemma 2.2. ⊡

Lemma 2.3

Let conditions 1.1 and 1.2 be fulfilled. Then

1. $K_1(t,x) > 0$ and $K_2(t,y) > 0$ for $t,\ x \in [0,\ 1]$ and $y \in [t_1,\ t_2]$.
2. $h(t) > 0,\ \varphi_1(t) > 0$ and $\varphi_2(t) > 0$ for $t \in [0,\ 1]$.

III. Main Results

Throughout this article, for $k = 0,\ 1, \cdots,$ we denote by $C^k[0,\ 1]$ the Banach space of all kth continuously differentiable functions $u(t)$ on $[0,\ 1]$ with the norm $\|u\| = \max_{t\in[0,\ 1]}\{|u(t)|,\ |u'(t)|,\ \cdots |u^k(t)|\}$ and let $E = C^2[0,\ 1]$. We denote by $L[0,\ 1]$ the Banach space of all integrable functions $u(t)$ on $[0,\ 1]$ with the norm $\|u\|_{L[0,\ 1]} = \int_0^1 |u(x)|\,dx.$

Theorem 3.1 (Existence)

Let conditions (1.1), (1.2) and (1.3) be fulfilled. Then the nonhomogeneous Sturn-Liouville boundary value problem has a unique positive solution $u_{\lambda_1,\lambda_2}(t)$ for all $\lambda_1 > 0$ and $\lambda_2 > 0$.

Proof

Let $K = \{u \in \mathbb{E}\ :\ u(t) \ge 0,\ 0 \le t \le 1\}$. Then K is a normal solid cone in $\mathbb{E}$ and his interior is defined by $\overset{\circ}{K} = \{u \in \mathbb{E}\ :\ u(t) > 0,\ 0 \le t \le 1\}$.

The rest of the proof is based on the following proposition

Proposition 3.1

Let $A_{\lambda_1,\lambda_2}\ :\ \overset{\circ}{K} \longrightarrow \overset{\circ}{K}$ an operator define for any $\lambda_1 > 0$ and $\lambda_2 > 0$ by :

$$A_{\lambda_1,\lambda_2}(u(t)) = \int_0^1 K_1(t,x)\int_{t_1}^{t_2} K_2(x,y)f(y,u(y),u''(y))\,dydx + h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t). \quad (3.1)$$

Then A_{λ_1,λ_2} is $r-$concave with $0 \le r < 1.$

Proof of proposition 3.1

Let $k \in [0, \ 1]$ and $u \in \mathring{K},$ it follows from (3.1)

$$A_{\lambda_1,\lambda_2}(ku(t)) = \int_0^1 K_1(t,x)\int_{t_1}^{t_2} K_2(x,y)f(y,ku(y),ku''(y))\,dydx + h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t). \quad (3.2)$$

By vertue the conditions (1.3), we obtain

$$A_{\lambda_1,\lambda_2}(ku(t)) \ge k^r \int_0^1 K_1(t,x)\int_{t_1}^{t_2} K_2(x,y)f(y,u(y),u''(y))\,dydx + h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t). (3.3)$$

Therefore, by inequality $h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t) \ge k^r\{h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t)\}$ and (3.3), we obtain

$$A_{\lambda_1,\lambda_2}(ku(t)) \ge k^r \left\{\int_0^1 K_1(t,x)\int_{t_1}^{t_2} K_2(x,y)f(y,u(y),u''(y))\,dydx + h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t)\right\}. \quad (3.4)$$

From (3.4), We conclude that

$$A_{\lambda_1,\lambda_2}(u(t)) \le k^r A_{\lambda_1,\lambda_2}(ku(t)). \quad (3.5)$$

Rest of proof of theorem (3.1)

It follows from lemma 2.1 and proposition 3.1 that A_{λ_1,λ_2} has a unique fixed point $u_{\lambda_1,\lambda_2} \in \mathring{K},$ which is the unique positive solution of the boundary value problem (1.1)-(1.5). This completes the proof. ⊡

Lemma 3.1

Under the conditions of Theorem (3.1) . The solution u_{λ_1,λ_2} of the boundary value problem (1.1) - (1.5) satisfies the following propertie :

$$\lim_{(\lambda_1,\lambda_2)\to(+\infty,+\infty)} \|u_{\lambda_1,\lambda_2}(t)\|_{\mathbb{E}} = +\infty$$

Proof

By virtue of the lemma 2.3, and the definition of $u_{\lambda_1,\lambda_2}(t)$:

$$\begin{aligned} u_{\lambda_1,\lambda_2}(t) &= A_{\lambda_1,\lambda_2}(u_{\lambda_1,\lambda_2}(t)) \\ &= \int_0^1 K_1(t,x)\int_{t_1}^{t_2} K_2(x,y)f(y,u_{\lambda_1,\lambda_2}(y),u''_{\lambda_1,\lambda_2}(y))\,dydx + h(t) + \lambda_1\varphi_1(t) + \lambda_2\varphi_2(t), \end{aligned}$$

we have

$$\lambda_1\varphi_1(t) + \lambda_2\varphi_2(t) \le \|u_{\lambda_1,\lambda_2}(t)\|_{\mathbb{E}}.$$

It is clear that $\lim_{(\lambda_1,\lambda_2)\to(+\infty,+\infty)} [\lambda_1\varphi_1(t) + \lambda_2\varphi_2(t)] = +\infty.$

From this last limit, we conclude :

$$\lim_{(\lambda_1,\lambda_2)\to(+\infty,+\infty)} \|u_{\lambda_1,\lambda_2}(t)\|_{\mathbb{E}} = +\infty.$$

The proof of lemma 3.1 is complete. ⊡

Theorem 3.2 (Continuous dependence)

Under the conditions of previous theorem . The solution u_{λ_1,λ_2} of the boundary value problem (1.1) - (1.5) , $u_{\lambda_1,\lambda_2}(t)$ is continuous in λ_1 and λ_2.

Proof

Let $(\lambda_1^0,\lambda_2^0)$ and $(\lambda_1^1,\lambda_2^1)$, such that $(0,0)<(\lambda_1^0,\lambda_2^0)<(\lambda_1^1,\lambda_2^1)(0<\lambda_1^0<\lambda_1^1$ and $0<\lambda_2^0<\lambda_2^1)$.

Put $\overline{n}=\left\{n>0\ ;\ u_{\lambda_1^0\lambda_2^0}(t)\leq nu_{\lambda_1^1\lambda_2^1}(t),\ t\in[0,1]\right\}$.

We assert that $\overline{n}\leq 1$.

Thus, we obtain $u_{\lambda_1^0\lambda_2^0}(t)\leq nu_{\lambda_1^1\lambda_2^1}(t)$, for $t\in[0,1]$.

Since $A_{\lambda_1\lambda_2}$ is strictly increasing in λ_1 and λ_2, we have

$$u_{\lambda_1^0\lambda_2^0}(t)=A_{\lambda_1^0\lambda_2^0}(u_{\lambda_1^0\lambda_2^0}(t))\leq A_{\lambda_1^0\lambda_2^0}(u_{\lambda_1^1\lambda_2^1}(t))<A_{\lambda_1^1\lambda_2^1}(u_{\lambda_1^1\lambda_2^1}(t))=u_{\lambda_1^1\lambda_2^1}(t)$$
$$u_{\lambda_1^0\lambda_2^0}(t)<u_{\lambda_1^1\lambda_2^1}(t),\ \text{for } t\in[0,\ 1]$$

It is easy to see that $u_{\lambda_1\lambda_2}(t)$ is also strictly increasing in λ_1 and λ_2.

For any $(\lambda_1^0,\ \lambda_2^0)>(0,0)$, we suppose $(\lambda_1,\lambda_2)\to(\lambda_1^0,\lambda_2^0)$, with $(\lambda_1^0,\lambda_2^0)<(\lambda_1,\lambda_2)$.

We have easily, $u_{\lambda_1^0\lambda_2^0}(t)<u_{\lambda_1\lambda_2}(t),\ t\in[0,1]$.

Put $\overline{m}=\left\{m>0,\ u_{\lambda_1\lambda_2}(t)\leq mu_{\lambda_1^0\lambda_2^0}(t),\ t\in[0,1]\right\}$

Then $\overline{m}\geq 1$, and $u_{\lambda_1^0\lambda_2^0}(t)\leq\frac{1}{m}u_{\lambda_1\lambda_2}(t)$ for $t\in[0,1]$.

Set $\Omega_{\lambda_1\lambda_2}=\min\left(\frac{\lambda_1}{\lambda_1^0},\ \frac{\lambda_2}{\lambda_2^0}\right)$.

That implies $\Omega_{\lambda_1\lambda_2}\geq 1$, and

$$u_{\lambda_1^0\lambda_2^0}(t)=A_{\lambda_1^0\lambda_2^0}(u_{\lambda_1^0\lambda_2^0}(t))\geq A_{\lambda_1^0\lambda_2^0}\left(\frac{1}{m}u_{\lambda_1\lambda_2}(t)\right)\tag{3.6}$$

$$A_{\lambda_1^0\lambda_2^0}\left(\frac{1}{m}u_{\lambda_1\lambda_2}(t)\right)>\frac{1}{\Omega_{\lambda_1\lambda_2}}A_{\lambda_1\lambda_2}\left(\frac{1}{m}(u_{\lambda_1\lambda_2}(t))\right)\tag{3.7}$$

$$A_{\lambda_1\lambda_2}\left(\frac{1}{m}u_{\lambda_1\lambda_2}(t)\right)\geq\frac{1}{m^r}A_{\lambda_1\lambda_2}\left(u_{\lambda_1\lambda_2}(t)\right)=\frac{1}{m^r}u_{\lambda_1\lambda_2}(t),\quad r,t\in[0,1]\tag{3.8}$$

Combining (3.6), (3.7) and (3.8), we can easily obtain :

$$u_{\lambda_1\lambda_2}(t)<\overline{m}^r\Omega_{\lambda_1\lambda_2}u_{\lambda_1^0\lambda_2^0}(t),\quad t\in[0,1].\tag{3.9}$$

Combining (3.9) and the definition of $\overline{m}$, it follows that

$$\overline{m}\leq\Omega_{\lambda_1\lambda_2}^{\frac{1}{1-r}},\ 0\leq r\leq 1.$$

And so

$$u_{\lambda_1\lambda_2}(t)\leq\overline{m}u_{\lambda_1^0\lambda_2^0}(t)\leq(\Omega_{\lambda_1\lambda_2}^{\frac{1}{1-r}}u_{\lambda_1^0\lambda_2^0}(t),\ 0\leq r\leq 1,\ 0\leq t\leq 1.\tag{3.10}$$

By virty of (3.10), we can write

$$\|u_{\lambda_1\lambda_2}(t) - u_{\lambda_1^0\lambda_2^0}(t)\| \leq (\Omega_{\lambda_1\lambda_2}^{\frac{1}{1-r}} - 1)\|u_{\lambda_1^0\lambda_2^0}(t)\|, \quad 0 \leq t \leq 1. \tag{3.11}$$

From (3.11) and the fact that $\lim_{(\lambda_1,\lambda_2)\to(\lambda_1^0,\lambda_2^0)} \Omega_{\lambda_1\lambda_2} = 1,$ it follows

$$\lim_{(\lambda_1,\lambda_2)\to(\lambda_1^0,\lambda_2^0)} \|u_{\lambda_1\lambda_2}(t) - u_{\lambda_1^0\lambda_2^0}(t)\| = 0. \tag{3.12}$$

Thus , finaly, $u_{\lambda_1,\lambda_2}(t)$ is continuous in λ_1 and λ_2. This complete the proof of theorem 3.2. ⊡

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